

Seesaw and leptogenesis: a triangular ansatz

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A triangular ansatz for the seesaw mechanism and baryogenesis via leptogenesis is explored. In a basis where both the charged lepton and the Majorana mass matrix are diagonal, the Dirac mass matrix can generally be written as the product of a unitary times a triangular matrix. We assume the unitary matrix to be the identity and then an upper triangular Dirac matrix. Constraints from bilarge lepton mixing and leptogenesis are studied.

I. INTRODUCTION

The baryogenesis via leptogenesis [1–4] is a framework proposed to explain the baryon asymmetry of the universe without grand unification, but works well even within the SO(10) model. It is so important because it is a consequence of the seesaw mechanism [5, 6], which in turn can explain the smallness of neutrino mass with respect to charged fermions.

In fact, the existence of very heavy Majorana neutrinos can generate small neutrino masses through mixing with Dirac neutrinos with masses similar to charged fermions (seesaw), and can produce a lepton asymmetry by means of decays to leptons and scalars (leptogenesis). Sphalerons convert roughly one half of this lepton asymmetry to a baryon asymmetry (baryogenesis).

In a previous paper [7], quark-lepton symmetry between Dirac mass matrices was considered together with diagonal and offdiagonal forms of the heavy Majorana mass matrix.

In the present paper, starting from a general formalism for mass matrices, we explore an ansatz leading to an upper triangular form for the Dirac neutrino mass matrix in a basis where both the charged lepton and the Majorana neutrino mass matrices are diagonal. In ref.[8] a lower triangular Dirac matrix is studied. This case is less predictive and the authors insert a texture zero.

II. THE SEESAW MECHANISM

According to the seesaw mechanism, the effective mass matrix of neutrinos is given by the formula

$$M_\nu \simeq M_D M_R^{-1} M_D^T, \quad (1)$$

where M_D is the Dirac mass matrix and M_R the Majorana mass matrix. For $M_R \gg M_D$, we have $M_\nu \ll M_D$.

In the standard model M_D is generated by the coupling with the same Higgs doublet that produces the up-quark mass matrix M_u , so that we expect the overall scale of M_D to be similar to the one of M_u . On the other hand, M_R is generated as a bare mass term (or by coupling to a Higgs singlet), so that its overall scale can be much larger than the weak scale.

III. LEPTOGENESIS

We consider leptogenesis formulas in the single-flavor approximation (see [9]). The baryon asymmetry, baryon to entropy fraction, is given by

$$Y_B \simeq \frac{1}{2} Y_L \quad (2)$$

and the lepton asymmetry by

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left(\frac{0.55 \cdot 10^{-3} eV}{\tilde{m}_1} \right)^{1.16} \quad (3)$$

in the strong washout regime, or

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left(\frac{\tilde{m}_1}{3.3 \cdot 10^{-3} eV} \right) \quad (4)$$

in the opposite weak washout regime. The parameter g_* is the number of light degrees of freedom, of the order $g_* \simeq 100$ in the standard case. Strong washout is realized for $\tilde{m}_1 \gg 3 \cdot 10^{-3}$, where $\tilde{m}_1 = (M_D^\dagger M_D)_{11}/M_1$.

Notice that Y_B is smaller than the baryon to photon ratio η by roughly a factor 7. The experimental value of the baryon asymmetry is (see [10]),

$$(Y_B)_{exp} \simeq 9 \cdot 10^{-11}. \quad (5)$$

The CP-violating asymmetry ϵ_1 , related to the decay of the lightest right-handed neutrino, is given by

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left(\frac{\text{Im}(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}(M_D^\dagger M_D)_{13}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_3} \right), \quad (6)$$

in the case $M_1 < M_2 \ll M_3$.

IV. THE TRIANGULAR MODEL

We take a basis where both the charged lepton and the right-handed neutrino mass matrices are diagonal:

$$M_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (7)$$

$$M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}. \quad (8)$$

The Dirac neutrino mass matrix can be written in the form $M_D = UY_\Delta v$, where $v = 175$ GeV is the v.e.v. of the Higgs doublet, U is a unitary matrix and Y_Δ is a triangular matrix

$$Y_\Delta = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ 0 & y_{22} & y_{23} \\ 0 & 0 & y_{33} \end{pmatrix}, \quad (9)$$

with real diagonal elements and complex mixing elements. The matrix U cancels out in unflavored leptogenesis but not in the seesaw formula.

Our fundamental ansatz consists in taking $U = 1$. Then the effective neutrino mass matrix becomes

$$M_\nu \simeq v^2 \begin{pmatrix} \frac{y_{11}^2}{M_1} + \frac{y_{12}^2}{M_2} + \frac{y_{13}^2}{M_3} & \frac{y_{12}y_{22}}{M_2} + \frac{y_{13}y_{23}}{M_3} & \frac{y_{13}y_{33}}{M_3} \\ * & \frac{y_{22}^2}{M_2} + \frac{y_{23}^2}{M_3} & \frac{y_{23}y_{33}}{M_3} \\ * & * & \frac{y_{33}^2}{M_3} \end{pmatrix}. \quad (10)$$

Parameters y are determined imposing the phenomenological structure of the effective matrix [11, 12]

$$M_\nu \simeq v_0 \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad (11)$$

with $\epsilon \simeq 0.05$, and $v_0 \simeq 0.02$. Assuming the largest y element to be of order 1, so that $v_0 \simeq v^2/M_3$, we obtain

$$y_{33} \sim 1, \quad y_{23} \sim 1, \quad y_{13} \sim \epsilon \quad (12)$$

and

$$y_{22} \sim \sqrt{(M_2/M_3)}, \quad y_{12} \sim \epsilon\sqrt{(M_2/M_3)}, \quad y_{11} \sim \epsilon\sqrt{(M_1/M_3)}. \quad (13)$$

The Dirac mass matrix becomes

$$M_D \simeq v \begin{pmatrix} \epsilon\sqrt{(M_1/M_3)} & \epsilon\sqrt{(M_2/M_3)} & \epsilon \\ 0 & \sqrt{(M_2/M_3)} & 1 \\ 0 & 0 & 1 \end{pmatrix}. \quad (14)$$

It is useful to compare this matrix with the up-quark matrix in a basis where it is triangular and the down-quark matrix is diagonal [13],

$$M_u \simeq m_t \begin{pmatrix} m_u/m_t & (m_c/m_t)V_{cd} & V_{td} \\ 0 & m_c/m_t & V_{ts} \\ 0 & 0 & 1 \end{pmatrix}, \quad (15)$$

showing deviation from quark-lepton symmetry in mass matrices.

We can now attempt to calculate the baryon asymmetry. First, we obtain the CP-asymmetry

$$\epsilon_1 = (3/16\pi)(\epsilon^2)(M_1/M_3)(\sin \alpha + \sin \gamma), \quad (16)$$

where α is the phase angle of y_{12} and γ the phase angle of y_{13} . Then, $\tilde{m}_1 \simeq 10^{-4}$, in the weak washout case. Finally, we get

$$Y_B \simeq 5 \cdot 10^{-5} \epsilon_1 \simeq 1.5 \cdot 10^{-4} (M_1/M_3)(\sin \alpha + \sin \gamma). \quad (17)$$

Matching with the value in (5), we obtain the relation

$$(M_1/M_3)(\sin \alpha + \sin \gamma) \simeq 6 \cdot 10^{-7} \quad (18)$$

and for $M_3 \simeq 10^{15}$ GeV also $(M_1)(\sin \alpha + \sin \gamma) \simeq 6 \cdot 10^8$ GeV.

V. CONCLUSION

Constraints from bilarge lepton mixing and leptogenesis determine a consistent relation among masses and phases in a triangular ansatz of the seesaw mechanism. Therefore, the rather strong condition $U = 1$ is nevertheless viable.

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- [1] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45
- [2] M. Luty, Phys. Rev. D 45 (1992) 455
- [3] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996) 169
- [4] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575 (2000) 61
- [5] P. Minkowski, Phys. Lett. B 67 (1977) 421
- [6] R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912
- [7] F. Buccella, D. Falcone and L. Oliver, Phys. Rev. D 77 (2008) 033002
- [8] G.C. Branco et al., Phys. Rev. D 67 (2003) 073025
- [9] A. Abada et al., hep-ph/0605281 and references therein
- [10] D.N. Spergel et al. [WMAP Collaboration], astro-ph/0603449
- [11] F. Vissani, J. High Energy Phys. 11 (1998) 025.
- [12] R. N. Mohapatra and W. Rodejohann, Phys. Rev. D 72 (2005) 053001
- [13] T.K. Kuo, S.W. Mansour and G.H. Wu, Phys. Rev. D 60 (1999) 093004